KBM 603

Final Exam

Instructions:

- Provide all necessary steps and be precise.
- Explain physical reasoning clearly where appropriate.

Q1

A resonantly driven two-level system exhibits population dynamics described by

$$P_e(t) = \sin^2\left(\frac{\Omega_R t}{2}\right).$$

The system is coupled to an environment causing loss of coherence.

(a) Explain the physical origin of Rabi oscillations.

(b) Describe how the measured signal changes when the dephasing time becomes much shorter than the Rabi period.

(c) How would the spectral linewidth change as the coherence time decreases? Explain the physical reason.

(d) Give one example from quantum technology where coherent oscillations are essential.

Q2

An ODMR spectrum of an NV-center ensemble is measured under varying magnetic field conditions.

(a) Explain the observed phenomena.

(b) Explain the physical origin of the magnetic-field-dependent splitting.

(c) Explain why finite splitting may still be observed at zero applied magnetic field.

(d) A shift of the ODMR center frequency is observed when the sample is heated. Explain the origin of this behavior.

(e) Explain the effect of applied microwave power dependence on the resonance linewidth.

Q3

A harmonic LC resonator has energy levels

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right).$$

A Josephson junction is introduced, producing the potential

$$U(\phi) = -E_J \cos\phi.$$

(a) Why are equally spaced energy levels unsuitable for qubit operation?

(b) Plot the energy levels for standard superconductor LC circuit and with Josephson junction. How does the Josephson cosine potential modify the energy spectrum compared to a LC?

(c) Plot energy leve diagrams for different E_J/E_C ratios. Which energy regime the transmons commonly operated?

(d) What limitation arises when the ratio E_J/E_C becomes extremely large?

Q4

Two indistinguishable single photons enter a 50:50 beam splitter from different input ports. The beam-splitter transformations are

$$\hat{a}^\dagger \to \frac{1}{\sqrt{2}}\left(\hat{c}^\dagger + i\hat{d}^\dagger\right),$$

$$\hat{b}^\dagger \to \frac{1}{\sqrt{2}}\left(i\hat{c}^\dagger + \hat{d}^\dagger\right).$$

The input state is

$$|1_a, 1_b\rangle = \hat{a}^\dagger\hat{b}^\dagger|0\rangle.$$

(a) Derive the output quantum state in terms of the output modes c and d.

(b) Determine the coefficient of the term

$$|1_c, 1_d\rangle.$$

(c) What does the result imply about coincidence detection at the output ports?

(d) Explain how partial distinguishability between the photons modifies the coincidence measurements.

(e) Discuss the importance of photon indistinguishability in photonic quantum technologies.

Q5

Weak coherent laser pulses are used in a BB84 quantum key distribution protocol.

(a) Explain the security vulnerability of using weak coherent laser pulses in BB84 protocol.

(b) A two-photon pulse is prepared in the state

$$|H\rangle|H\rangle.$$

Describe how a photon-number-splitting attack can extract information from this state.

(c) Explain the operating principle of the decoy-state protocol.

(d) A qubit is prepared in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right).$$

Calculate the probabilities of obtaining $|H\rangle$ and $|V\rangle$ when measured in the $\{|H\rangle, |V\rangle\}$ basis.

(e) Explain why measurement in a non-matching basis disturbs the transmitted quantum state.