Fabry–Perot Cavities and Microring Resonators

Unified Cavity Theory

1 Introduction

Optical cavities are fundamental building blocks of photonics and quantum technologies. Two of the most important realizations are:

- Fabry–Perot cavities,
- microring resonators.

Although their geometries are different, both are governed by the same physical principle:

> Cavity resonance occurs when the optical field reproduces itself after one round trip. Equivalently, the round-trip phase must be an integer multiple of 2π.

The central idea is that many delayed copies of the optical field interfere. At resonance they interfere constructively, leading to field enhancement. Away from resonance they interfere destructively, suppressing transmission or circulating power.

2 Fabry–Perot Cavity: Basic Geometry

A Fabry–Perot cavity consists of two partially reflecting mirrors separated by a distance L.

Let the incident field be

$$E_{\text{in}}(t) = E_0 e^{-i\omega t}. \tag{1}$$

The mirrors are described by amplitude reflection and transmission coefficients:

$$r_1,\ r_2,\ t_1,\ t_2. \tag{2}$$

The corresponding intensity reflectivities and transmissivities are

$$R_i = |r_i|^2, \qquad T_i = |t_i|^2. \tag{3}$$

For lossless mirrors,

$$R_i + T_i = 1. \tag{4}$$

The propagation constant inside the cavity is

$$k = \frac{n\omega}{c} = \frac{2\pi n}{\lambda}. \tag{5}$$

A single pass through the cavity gives phase

$$kL. \tag{6}$$

A round trip gives phase

$$\phi_{\text{FP}} = 2kL. \tag{7}$$

Using $k = 2\pi n/\lambda$,

$$\phi_{\text{FP}} = \frac{4\pi nL}{\lambda}. \tag{8}$$

3 Fabry–Perot Resonance Condition

Constructive interference occurs when

$$\phi_{\text{FP}} = 2\pi m, \qquad m \in \mathbb{Z}. \tag{9}$$

Therefore,

$$2kL = 2\pi m. \tag{10}$$

Using $k = 2\pi n/\lambda$,

$$2nL = m\lambda. \tag{11}$$

Equivalently, in frequency,

$$\nu_m = \frac{mc}{2nL}. \tag{12}$$

The integer m is the longitudinal mode number of the Fabry–Perot cavity.

4 Fabry–Perot Transmission: Field Derivation

The transmitted field is the coherent sum of all fields that leak out through the second mirror.

The first transmitted contribution enters the cavity through mirror 1, propagates once through the cavity, and exits mirror 2:

$$E_0^{(0)} = t_1 t_2 e^{ikL} E_{\text{in}}. \tag{13}$$

The next contribution makes one additional round trip before exiting:

$$E_0^{(1)} = t_1 t_2 r_1 r_2 e^{ikL} e^{i2kL} E_{\text{in}}. \tag{14}$$

The next contribution makes two additional round trips:

$$E_0^{(2)} = t_1 t_2 (r_1 r_2)^2 e^{ikL} e^{i4kL} E_{\text{in}}. \tag{15}$$

Therefore, the total transmitted field is

$$E_{\text{trans}} = t_1 t_2 e^{ikL} E_{\text{in}} \left[1 + r_1 r_2 e^{i2kL} + (r_1 r_2)^2 e^{i4kL} + \cdots \right]. \tag{16}$$

Using the round-trip phase

$$\phi = 2kL, \tag{17}$$

we get

$$E_{\text{trans}} = t_1 t_2 e^{ikL} E_{\text{in}} \sum_{q=0}^{\infty} \left(r_1 r_2 e^{i\phi} \right)^q. \tag{18}$$

This is a geometric series:

$$\sum_{q=0}^{\infty} x^q = \frac{1}{1-x}, \qquad |x| < 1. \tag{19}$$

Therefore,

$$E_{\text{trans}} = \frac{t_1 t_2 e^{ikL}}{1 - r_1 r_2 e^{i\phi}} E_{\text{in}}. \tag{20}$$

The intensity transmission is

$$T_{\text{FP}} = \left| \frac{E_{\text{trans}}}{E_{\text{in}}} \right|^2. \tag{21}$$

Thus,

$$T_{\text{FP}} = \left| \frac{t_1 t_2}{1 - r_1 r_2 e^{i\phi}} \right|^2. \tag{22}$$

The factor e^{ikL} disappears because its magnitude is one.

5 Airy Function for Identical Lossless Mirrors

For identical mirrors,

$$R_1 = R_2 = R, \qquad T_1 = T_2 = 1 - R. \tag{23}$$

Assuming real amplitude coefficients,

$$r_1 r_2 = R, \qquad t_1 t_2 = 1 - R. \tag{24}$$

Then

$$T_{\text{FP}} = \left| \frac{1 - R}{1 - Re^{i\phi}} \right|^2 . \tag{25}$$

The denominator magnitude is

$$|1 - Re^{i\phi}|^2 = (1 - Re^{i\phi})(1 - Re^{-i\phi}) \tag{26}$$
$$= 1 - Re^{i\phi} - Re^{-i\phi} + R^2 \tag{27}$$
$$= 1 + R^2 - R(e^{i\phi} + e^{-i\phi}). \tag{28}$$

Using

$$e^{i\phi} + e^{-i\phi} = 2\cos\phi, \tag{29}$$

we obtain

$$|1 - Re^{i\phi}|^2 = 1 + R^2 - 2R\cos\phi. \tag{30}$$

Therefore,

$$T_{\text{FP}}(\phi) = \frac{(1 - R)^2}{1 + R^2 - 2R\cos\phi}. \tag{31}$$

This is the Fabry–Perot Airy transmission function.

6 Alternative Airy Form

Using the trigonometric identity

$$1 - \cos\phi = 2\sin^2\left(\frac{\phi}{2}\right), \tag{32}$$

the denominator becomes

$$1 + R^2 - 2R\cos\phi = (1-R)^2 + 2R(1-\cos\phi) \tag{33}$$
$$= (1-R)^2 + 4R\sin^2\left(\frac{\phi}{2}\right). \tag{34}$$

Thus,

$$T_{\mathrm{FP}}(\phi) = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(\phi/2)}. \tag{35}$$

Dividing numerator and denominator by $(1-R)^2$ gives

$$T_{\mathrm{FP}}(\phi) = \frac{1}{1 + F\sin^2(\phi/2)}, \tag{36}$$

where

$$F = \frac{4R}{(1-R)^2}. \tag{37}$$

The quantity F is the coefficient of finesse. It should not be confused with the cavity finesse $\mathcal{F}$.

At resonance,

$$\phi = 2\pi m, \tag{38}$$

so

$$\sin^2(\phi/2) = 0. \tag{39}$$

Therefore,

$$T_{\mathrm{FP}} = 1 \tag{40}$$

for identical, lossless mirrors.

Even if each mirror is highly reflective, the on-resonance transmission can be unity for a symmetric, lossless Fabry–Perot cavity. This occurs because the multiple transmitted fields interfere constructively in the forward direction.

7 Fabry–Perot Reflection

The reflected field is also a coherent sum. It contains the directly reflected field from the first mirror and the field leaking back out of the cavity.

The result for the reflected field can be written as

$$\frac{E_{\text{refl}}}{E_{\text{in}}} = \frac{r_1 - r_2 e^{i\phi}}{1 - r_1 r_2 e^{i\phi}}, \tag{41}$$

up to phase convention.

For identical mirrors,

$$r_1 = r_2 = r, \tag{42}$$

and at resonance,

$$e^{i\phi} = 1. \tag{43}$$

Therefore,

$$E_{\text{refl}} = 0. \tag{44}$$

At resonance, a symmetric lossless Fabry–Perot cavity transmits all incident power and reflects no power.

This is directly analogous to critical coupling in microring resonators.

8 Fabry–Perot Linewidth and Finesse

The resonance peaks become sharper as R increases.

The free spectral range is

$$\Delta\nu_{\text{FSR}}^{\text{FP}} = \frac{c}{2n_g L}. \tag{45}$$

The finesse is defined as

$$\mathcal{F} = \frac{\Delta\nu_{\text{FSR}}}{\delta\nu}, \tag{46}$$

where $\delta\nu$ is the full width at half maximum.

For a high-reflectivity, lossless Fabry–Perot cavity,

$$\mathcal{F} \approx \frac{\pi\sqrt{R}}{1 - R}. \tag{47}$$

The linewidth is therefore

$$\delta\nu = \frac{\Delta\nu_{\text{FSR}}}{\mathcal{F}}. \tag{48}$$

9 Fabry–Perot Intracavity Field Build-Up

The intracavity forward field just after entering through mirror 1 is

$$E_{\text{cav}} = t_1 E_{\text{in}} \sum_{q=0}^{\infty} \left(r_1 r_2 e^{i\phi} \right)^q . \tag{49}$$

Thus,

$$E_{\text{cav}} = \frac{t_1}{1 - r_1 r_2 e^{i\phi}} E_{\text{in}}. \tag{50}$$

At resonance,

$$E_{\text{cav,res}} = \frac{t_1}{1 - r_1 r_2} E_{\text{in}}. \tag{51}$$

For identical lossless mirrors,

$$t_1 = \sqrt{1-R}, \qquad r_1 r_2 = R. \tag{52}$$

Therefore,

$$\left| \frac{E_{\text{cav,res}}}{E_{\text{in}}} \right|^2 = \frac{1-R}{(1-R)^2} = \frac{1}{1-R}. \tag{53}$$

So the intracavity intensity enhancement is

$$B_{\text{FP}} = \frac{I_{\text{cav}}}{I_{\text{in}}} = \frac{1}{1-R}. \tag{54}$$

The transmitted intensity can be unity while the intracavity intensity is much larger than the incident intensity. This is not a violation of energy conservation because the cavity stores energy over many round trips.

10 Photon Lifetime and Quality Factor

The photon lifetime τ_{ph} is the characteristic time for stored energy to decay.

If the stored energy is $U(t)$,

$$U(t) = U_0 e^{-t/\tau_{\text{ph}}}. \tag{55}$$

The linewidth in angular frequency is related to the photon lifetime by

$$\delta\omega = \frac{1}{\tau_{\text{ph}}}. \tag{56}$$

In ordinary frequency,

$$\delta\nu = \frac{1}{2\pi\tau_{\text{ph}}}. \tag{57}$$

The quality factor is

$$Q = \frac{\omega_0}{\delta\omega} = \omega_0\tau_{\text{ph}}. \tag{58}$$

Equivalently,

$$Q = \frac{\nu_0}{\delta\nu}. \tag{59}$$

11 Microring Resonator: Geometry

A microring resonator is a closed-loop waveguide coupled to one or more bus waveguides. For a ring of radius R,

$$L_{\text{rt}} = 2\pi R. \tag{60}$$

The guided mode propagation constant is

$$\beta(\omega) = \frac{n_{\text{eff}}(\omega)\omega}{c}. \tag{61}$$

The round-trip phase is

$$\phi_{\text{MRR}} = \beta L_{\text{rt}}. \tag{62}$$

12 Microring Resonance Condition

The resonance condition is

$$\phi_{\text{MRR}} = 2\pi m. \tag{63}$$

Therefore,

$$\beta L_{\text{rt}} = 2\pi m. \tag{64}$$

Using

$$\beta = \frac{2\pi n_{\text{eff}}}{\lambda}, \tag{65}$$

we get

$$n_{\text{eff}} L_{\text{rt}} = m\lambda. \tag{66}$$

The integer m is the azimuthal mode number of the microring resonator.

13 Mapping Between Fabry–Perot and Microring Cavities

The Fabry–Perot resonance condition is

$$2nL = m\lambda. \tag{67}$$

The microring resonance condition is

$$n_{\text{eff}} L_{\text{rt}} = m\lambda. \tag{68}$$

Therefore, the correspondence is

$$2L \longleftrightarrow L_{\text{rt}}. \tag{69}$$

A microring resonator may be understood as a Fabry–Perot cavity folded into a closed loop, where the round-trip length is the ring circumference.

14 Microring Free Spectral Range

The group index is

$$n_g = n_{\text{eff}} - \lambda \frac{dn_{\text{eff}}}{d\lambda}. \tag{70}$$

The round-trip group delay is

$$\tau_{\text{rt}} = \frac{d\phi}{d\omega}. \tag{71}$$

For a microring,

$$\tau_{\text{rt}} = \frac{n_g L_{\text{rt}}}{c}. \tag{72}$$

The free spectral range is the inverse of the round-trip group delay:

$$\Delta\nu_{\text{FSR}}^{\text{MRR}} = \frac{1}{\tau_{\text{rt}}}. \tag{73}$$

Thus,

$$\Delta\nu_{\text{FSR}}^{\text{MRR}} = \frac{c}{n_g L_{\text{rt}}}. \tag{74}$$

Using $L_{\text{rt}} = 2\pi R$,

$$\Delta\nu_{\mathrm{FSR}}^{\mathrm{MRR}} = \frac{c}{n_g 2\pi R}. \tag{75}$$

15 Microring Coupler Model

Consider a single-bus all-pass microring.

Let

- t be the self-coupling amplitude coefficient,
- κ be the cross-coupling amplitude coefficient,
- a be the round-trip amplitude transmission due to intrinsic propagation loss,
- ϕ be the round-trip phase.

For a lossless directional coupler,

$$|t|^2 + |\kappa|^2 = 1. \tag{76}$$

The intrinsic round-trip amplitude factor is

$$a = e^{-\alpha L_{\mathrm{rt}}/2}, \tag{77}$$

where α is the power attenuation coefficient.

Therefore, after one round trip, the field acquires the factor

$$ae^{i\phi}. \tag{78}$$

16 Microring Field Build-Up

The field coupled into the ring is proportional to

$$i\kappa E_{\mathrm{in}}. \tag{79}$$

After one round trip, the circulating field is multiplied by

$$ate^{i\phi}. \tag{80}$$

Therefore, the ring field is the geometric sum

$$E_{\mathrm{ring}} = i\kappa E_{\mathrm{in}} \left[1 + ate^{i\phi} + (ate^{i\phi})^2 + \cdots\right]. \tag{81}$$

Thus,

$$E_{\mathrm{ring}} = \frac{i\kappa}{1 - ate^{i\phi}} E_{\mathrm{in}}. \tag{82}$$

This is directly analogous to the Fabry–Perot intracavity field:

$$E_{\text{cav}} = \frac{t_1}{1 - r_1 r_2 e^{i\phi}} E_{\text{in}}. \tag{83}$$

17 Microring Through-Port Transmission

The through-port field is the coherent sum of:

- the directly transmitted bus field,
- the field coupled back from the ring.

For a single-bus all-pass ring, the field transmission is

$$\frac{E_{\text{through}}}{E_{\text{in}}} = \frac{t - a e^{i\phi}}{1 - a t e^{i\phi}}. \tag{84}$$

The power transmission is

$$T_{\text{through}} = \left| \frac{t - a e^{i\phi}}{1 - a t e^{i\phi}} \right|^2. \tag{85}$$

Expanding the magnitude gives

$$T_{\text{through}} = \frac{t^2 + a^2 - 2at\cos\phi}{1 + a^2 t^2 - 2at\cos\phi}. \tag{86}$$

This is the microring analogue of the Airy transmission function.

18 Microring Resonance Dip and Critical Coupling

At resonance,

$$e^{i\phi} = 1. \tag{87}$$

Therefore,

$$T_{\text{res}} = \left| \frac{t - a}{1 - at} \right|^2. \tag{88}$$

Critical coupling occurs when

$$t = a. \tag{89}$$

Then,

$$T_{\text{res}} = 0. \tag{90}$$

Critical coupling means that the coupling loss equals the intrinsic cavity loss. At this point, destructive interference cancels the through-port output at resonance.

19 Microring Coupling Regimes

The coupling regime is determined by comparing t and a.

19.1 Undercoupled Regime

$$t > a. \tag{91}$$

The coupling is weak compared to internal loss.

19.2 Critical Coupling

$$t = a. \tag{92}$$

The resonance dip reaches zero transmission in the ideal case.

19.3 Overcoupled Regime

$$t < a. \tag{93}$$

The coupling is stronger than intrinsic loss.

In terms of quality factors, overcoupling means that the coupling quality factor is smaller than the intrinsic quality factor.

20 Intrinsic, Coupling, and Loaded Quality Factors

The total cavity decay rate is

$$\kappa_{\text{tot}} = \kappa_i + \kappa_c, \tag{94}$$

where

- κ_i is the intrinsic loss rate,
- κ_c is the coupling loss rate.

The loaded quality factor is

$$Q_L = \frac{\omega_0}{\kappa_{\text{tot}}}. \tag{95}$$

Similarly,

$$Q_i = \frac{\omega_0}{\kappa_i}, \qquad Q_c = \frac{\omega_0}{\kappa_c}. \tag{96}$$

Therefore,

$$\frac{1}{Q_L} = \frac{1}{Q_i} + \frac{1}{Q_c}. \tag{97}$$

Equivalently,

$$Q_L = \frac{Q_i Q_c}{Q_i + Q_c}. \tag{98}$$

The coupling regimes become:

$$Q_c > Q_i \qquad \text{undercoupled,} \tag{99}$$

$$Q_c = Q_i \qquad \text{critical coupling,} \tag{100}$$

$$Q_c < Q_i \qquad \text{overcoupled.} \tag{101}$$

21 Linewidth and Lorentzian Approximation

Near a resonance frequency ω_0, the phase can be expanded as

$$\phi(\omega) \approx \phi(\omega_0) + \left.\frac{d\phi}{d\omega}\right|_{\omega_0} (\omega - \omega_0). \tag{102}$$

At resonance,

$$\phi(\omega_0) = 2\pi m. \tag{103}$$

Therefore,

$$\phi(\omega) \approx 2\pi m + \tau_{\text{rt}}(\omega - \omega_0). \tag{104}$$

Close to resonance, the cavity response takes the Lorentzian form

$$H(\omega) \propto \frac{1}{i(\omega - \omega_0) + \kappa_{\text{tot}}/2}. \tag{105}$$

The intensity response is

$$|H(\omega)|^2 \propto \frac{1}{(\omega - \omega_0)^2 + (\kappa_{\text{tot}}/2)^2}. \tag{106}$$

The full width at half maximum in angular frequency is

$$\delta\omega = \kappa_{\text{tot}}. \tag{107}$$

Thus,

$$Q_L = \frac{\omega_0}{\delta\omega}. \tag{108}$$

22 Finesse and Effective Number of Round Trips

For both Fabry–Perot and microring cavities, finesse is defined as

$$\mathcal{F} = \frac{\Delta\nu_{\text{FSR}}}{\delta\nu}. \tag{109}$$

The effective number of round trips is approximately

$$N_{\text{rt}} \sim \frac{\mathcal{F}}{\pi}. \tag{110}$$

Thus, the effective interaction length in a ring is

$$L_{\text{eff}} \sim N_{\text{rt}} L_{\text{rt}}. \tag{111}$$

High finesse means the field remains in the cavity for many round trips, producing strong resonant enhancement.

23 Unified Cavity Response

Both Fabry–Perot and microring cavities share the same mathematical structure:

$$\text{cavity response} \sim \frac{1}{1 - \rho e^{i\phi}}, \tag{112}$$

where ρ is the round-trip amplitude factor.

For a Fabry–Perot cavity,

$$\rho = r_1 r_2. \tag{113}$$

For a microring resonator,

$$\rho = at. \tag{114}$$

The denominator $1 - \rho e^{i\phi}$ is the mathematical signature of multiple round-trip interference.

24 Comparison Table

Quantity	Fabry–Perot cavity	Microring resonator
Round-trip length	$2L$	$L_{\mathrm{rt}} = 2\pi R$
Round-trip phase	$2kL$	βL_{rt}
Resonance condition	$2nL = m\lambda$	$n_{\mathrm{eff}} L_{\mathrm{rt}} = m\lambda$
FSR	$\frac{c}{2n_g L}$	$\frac{c}{n_g L_{\mathrm{rt}}}$
Feedback mechanism	Mirror reflection	Circulation in ring
Coupling mechanism	Mirror transmission	Directional coupler
Response denominator	$1 - r_1 r_2 e^{i\phi}$	$1 - ate^{i\phi}$
Linewidth	$\delta\omega$	$\kappa_i + \kappa_c$
Quality factor	$Q = \omega_0/\delta\omega$	$1/Q_L = 1/Q_i + 1/Q_c$

Table 1: Mapping between Fabry–Perot cavities and microring resonators.

25 Physical Summary

Fabry–Perot cavities and microring resonators are different geometrical realizations of the same cavity physics.

A Fabry–Perot cavity stores light by repeated reflection between two mirrors.

A microring resonator stores light by repeated circulation inside a closed waveguide.

In both systems:

- the optical field accumulates phase in one round trip,
- resonance occurs when the round-trip phase is $2\pi m$,
- the transmitted and stored fields arise from a geometric series,
- linewidth is determined by the total energy decay rate,
- finesse measures the sharpness of resonance,
- quality factor measures the energy storage capability.

The universal passive cavity response is

$$\frac{1}{1 - \rho e^{i\phi}}.$$

This expression connects Fabry–Perot cavities, microring resonators, and many other optical resonators.