Photonic Integrated Circuits — Lecture Notes (Next Lecture)
FDTD Connection to Maxwell; Yee Cell & Boundary Conditions;
Lumerical MODE (FDE) Eigenmode Solver and Sparse-Matrix Formulation

Contents

1 Learning goals and connection to previous lecture

In the previous lecture you derived:

- Maxwell curl equations in time and frequency domain,
- wave equation / Helmholtz equation for monochromatic fields,
- plane-wave and evanescent-wave solutions.

This lecture connects those continuous equations to:

- **FDTD**: explicit time stepping of Maxwell curl equations on a staggered grid (Yee cell),
- **Boundary conditions**: how to truncate open domains and model conductors/material interfaces,
- **MODE/FDE**: frequency-domain finite-difference eigenvalue formulation for guided modes, solved with sparse-matrix techniques.

Throughout we use SI units.

2 EM fields and the FDTD connection (and why it is preferred)

2.1 Start from time-domain Maxwell curl equations

In a general linear medium (possibly lossy), write

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \mathbf{J}(\mathbf{r}, t) + \frac{\partial \mathbf{D}}{\partial t}. \tag{2}$$

Constitutive relations for a simple isotropic medium:

$$\mathbf{D} = \varepsilon \mathbf{E}, \qquad \mathbf{B} = \mu \mathbf{H}. \tag{3}$$

For conduction current in an ohmic medium:

$$\mathbf{J} = \sigma \mathbf{E}. \tag{4}$$

Then (2) becomes

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}. \tag{5}$$

2.2 Core idea of FDTD

Look at (1)–(5):

time-derivative of $\mathbf{H}$ depends on spatial curl of $\mathbf{E}$, time-derivative of $\mathbf{E}$ depends on spatial curl of $\mathbf{H}$.

FDTD discretizes:

- **Space**: finite differences on a grid (Yee lattice),
- **Time**: finite differences with a **leapfrog** scheme (E and H staggered in time),
- **Marching**: update fields explicitly step-by-step (no global matrix solve per time step in standard Yee FDTD).

The canonical Yee scheme uses centered differences on a staggered grid in space and time for each field component, which is the defining novelty of Yee's 1966 formulation.

2.3 Why FDTD is preferred in many PIC problems

For integrated photonics you frequently need:

- Broadband responses (transmission/reflection spectra) from one simulation by pulsed excitation,
- Complex geometries (bends, gratings, subwavelength patterning) and strong index contrast,
- Near-to-far-field transformations (grating couplers, antennas),
- Time-domain dynamics (ring resonator transients, pulse propagation, switching).

FDTD naturally provides time evolution and broadband spectra from a single run, and it directly solves Maxwell's curl equations on a grid . It also interfaces cleanly with absorbing boundaries (PML) that mimic open space .

2.4 Warm-up: 1D FDTD derivation from Maxwell (full steps)

To see the mechanics, derive the 1D update equations explicitly.

2.4.1 1D wave traveling along x with E_z, H_y

Assume fields depend only on x and t and choose polarization:

$$\mathbf{E} = \hat{\mathbf{z}} E_z(x,t), \qquad \mathbf{H} = \hat{\mathbf{y}} H_y(x,t).$$

Compute curls. In Cartesian coordinates,

$$\nabla \times \mathbf{E} = \begin{pmatrix} \partial_y E_z - \partial_z E_y \\ \partial_z E_x - \partial_x E_z \\ \partial_x E_y - \partial_y E_x \end{pmatrix}.$$

Here $E_x = E_y = 0$ and no y, z dependence, so

$$\nabla \times \mathbf{E} = \begin{pmatrix} 0 \\ -\partial_x E_z \\ 0 \end{pmatrix} = -\hat{\mathbf{y}} \frac{\partial E_z}{\partial x}.$$

Also $\mathbf{B} = \mu \mathbf{H} = \hat{\mathbf{y}} \mu H_y$. Then (1) becomes

$$-\hat{\mathbf{y}} \frac{\partial E_z}{\partial x} = -\frac{\partial}{\partial t}(\hat{\mathbf{y}} \mu H_y) \Rightarrow \frac{\partial E_z}{\partial x} = \mu \frac{\partial H_y}{\partial t}.$$

Thus

$$\boxed{\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \frac{\partial E_z}{\partial x}.} \tag{6}$$

Now compute $\nabla \times \mathbf{H}$. With $\mathbf{H} = \hat{\mathbf{y}} H_y(x, t)$,

$$\nabla \times \mathbf{H} = \begin{pmatrix} \partial_y H_z - \partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \partial_x H_y \end{pmatrix} = \hat{\mathbf{z}} \frac{\partial H_y}{\partial x}.$$

In lossless medium $\sigma = 0$, $\mathbf{D} = \varepsilon \mathbf{E} = \hat{\mathbf{z}} \varepsilon E_z$. Then (2) gives

$$\hat{\mathbf{z}} \frac{\partial H_y}{\partial x} = \frac{\partial}{\partial t}(\hat{\mathbf{z}} \varepsilon E_z) \Rightarrow \frac{\partial H_y}{\partial x} = \varepsilon \frac{\partial E_z}{\partial t}.$$

Hence

$$\boxed{\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \frac{\partial H_y}{\partial x}.} \tag{7}$$

2.4.2 Discretize in space and time (centered differences)

Let spatial grid index be i with step Δx, and time index be n with step Δt. Yee staggering in 1D places:

$$E_z \text{ at integer } i, \text{ integer } n; \qquad H_y \text{ at half-integer } i + \tfrac{1}{2}, \text{ half-time } n + \tfrac{1}{2}.$$

Define:

$$E_z^n(i) \approx E_z(i\Delta x, n\Delta t), \qquad H_y^{n+\frac{1}{2}}\left(i + \frac{1}{2}\right) \approx H_y\Big((i + \tfrac{1}{2})\Delta x, (n + \tfrac{1}{2})\Delta t\Big).$$

Approximate derivatives using centered differences consistent with staggering.

For (6) at $(i + \frac{1}{2}, n + \frac{1}{2})$:

$$\left.\frac{\partial H_y}{\partial t}\right|_{i+\frac{1}{2}}^{n+\frac{1}{2}} \approx \frac{H_y^{n+\frac{1}{2}}(i + \frac{1}{2}) - H_y^{n-\frac{1}{2}}(i + \frac{1}{2})}{\Delta t},$$

$$\left.\frac{\partial E_z}{\partial x}\right|_{i+\frac{1}{2}}^{n} \approx \frac{E_z^n(i+1) - E_z^n(i)}{\Delta x}.$$

Thus

$$\frac{H_y^{n+\frac{1}{2}}(i + \frac{1}{2}) - H_y^{n-\frac{1}{2}}(i + \frac{1}{2})}{\Delta t} = \frac{1}{\mu} \frac{E_z^n(i+1) - E_z^n(i)}{\Delta x}.$$

Solve for the new magnetic field:

$$\boxed{H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2}\right) = H_y^{n-\frac{1}{2}}\left(i+\frac{1}{2}\right) + \frac{\Delta t}{\mu \Delta x}\left[E_z^n(i+1) - E_z^n(i)\right].} \tag{8}$$

Similarly, discretize (7) at $(i, n+1)$ using H at half steps:

$$\left.\frac{\partial E_z}{\partial t}\right|_i^{n+\frac{1}{2}} \approx \frac{E_z^{n+1}(i) - E_z^n(i)}{\Delta t},$$

$$\left.\frac{\partial H_y}{\partial x}\right|_i^{n+\frac{1}{2}} \approx \frac{H_y^{n+\frac{1}{2}}(i+\frac{1}{2}) - H_y^{n+\frac{1}{2}}(i-\frac{1}{2})}{\Delta x}.$$

Hence

$$\frac{E_z^{n+1}(i) - E_z^n(i)}{\Delta t} = \frac{1}{\varepsilon}\frac{H_y^{n+\frac{1}{2}}(i+\frac{1}{2}) - H_y^{n+\frac{1}{2}}(i-\frac{1}{2})}{\Delta x},$$

so

$$\boxed{E_z^{n+1}(i) = E_z^n(i) + \frac{\Delta t}{\varepsilon \Delta x}\left[H_y^{n+\frac{1}{2}}(i+\frac{1}{2}) - H_y^{n+\frac{1}{2}}(i-\frac{1}{2})\right].} \tag{9}$$

Equations (8) and (9) are the archetypal leapfrog FDTD updates.

2.5 From 1D to 3D: general Yee update structure

In 3D, the same logic applies component-wise to the six coupled curl equations. The Yee scheme staggers:

- Electric field components on cell **edges**,
- Magnetic field components on cell **faces**,
- E and H **half-step staggered in time** (leapfrog).

This staggered placement is the essence of the Yee lattice.

2.6 Stability (CFL condition) and numerical dispersion

Because the standard Yee update is explicit, stability requires the time step satisfy a Courant–Friedrichs–Lewy (CFL) condition. In 3D Cartesian grid with steps $\Delta x, \Delta y, \Delta z$, the standard bound is

$$\boxed{\Delta t \le \frac{1}{c}\frac{1}{\sqrt{\left(\frac{1}{\Delta x^2}\right) + \left(\frac{1}{\Delta y^2}\right) + \left(\frac{1}{\Delta z^2}\right)}},} \tag{10}$$

where $c = 1/\sqrt{\mu\varepsilon}$ in the fastest medium present (often c_0 in free space). Violation produces exponential growth (unstable fields). Even when stable, finite grid causes **numerical dispersion**: the phase velocity depends on direction and frequency relative to the grid.

3 Yee cell and boundary conditions (PML, metal, and common PIC needs)

3.1 Yee cell geometry and indexing

3.1.1 Field locations

Consider a cubic cell indexed by (i, j, k) with steps $\Delta x, \Delta y, \Delta z$.

A common convention:

- E_x located at $(i + \frac{1}{2}, j, k)$ (edge parallel to x),
- E_y located at $(i, j + \frac{1}{2}, k)$,
- E_z located at $(i, j, k + \frac{1}{2})$,
- H_x located at $(i, j + \frac{1}{2}, k + \frac{1}{2})$ (face normal to x),
- H_y located at $(i + \frac{1}{2}, j, k + \frac{1}{2})$,
- H_z located at $(i + \frac{1}{2}, j + \frac{1}{2}, k)$.

This staggering ensures each discrete curl uses neighboring components centered around the update point, yielding second-order accuracy and good energy behavior (a key reason Yee's arrangement is robust).

3.1.2 Example: discrete curl for updating E_x

From Maxwell:

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J}.$$

Assume $\mathbf{D} = \varepsilon \mathbf{E}$ and $\mathbf{J} = \sigma \mathbf{E}$ (simple ohmic loss). Then component-wise:

$$\varepsilon \frac{\partial E_x}{\partial t} = (\nabla \times \mathbf{H})_x - \sigma E_x, \qquad (\nabla \times \mathbf{H})_x = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}.$$

Discretize at E_x location $(i + \frac{1}{2}, j, k)$ and time n:

$$\left.\frac{\partial H_z}{\partial y}\right|_{i+\frac{1}{2},j,k} \approx \frac{H_z(i + \frac{1}{2}, j + \frac{1}{2}, k) - H_z(i + \frac{1}{2}, j - \frac{1}{2}, k)}{\Delta y},$$

$$\left.\frac{\partial H_y}{\partial z}\right|_{i+\frac{1}{2},j,k} \approx \frac{H_y(i + \frac{1}{2}, j, k + \frac{1}{2}) - H_y(i + \frac{1}{2}, j, k - \frac{1}{2})}{\Delta z}.$$

For time, with leapfrog (say H at $n + \frac{1}{2}$ and E at n), use

$$\left.\frac{\partial E_x}{\partial t}\right|^{n+\frac{1}{2}} \approx \frac{E_x^{n+1} - E_x^n}{\Delta t}.$$

A standard lossy-update rearrangement (explicit) is obtained by writing

$$\varepsilon \frac{E_x^{n+1} - E_x^n}{\Delta t} = (\nabla \times \mathbf{H})_x^{n+\frac{1}{2}} - \sigma \frac{E_x^{n+1} + E_x^n}{2},$$

where the conductivity term is centered in time (improves stability/accuracy). Solve for E_x^{n+1}:

$$\varepsilon \frac{E_x^{n+1} - E_x^n}{\Delta t} + \sigma \frac{E_x^{n+1} + E_x^n}{2} = (\nabla \times \mathbf{H})_x^{n+\frac{1}{2}} .$$

Collect E_x^{n+1} terms:

$$\left(\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2} \right) E_x^{n+1} = \left(\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2} \right) E_x^n + (\nabla \times \mathbf{H})_x^{n+\frac{1}{2}} .$$

Thus

$$\boxed{\begin{aligned} E_x^{n+1} = C_e\, E_x^n + C_h \Bigg[& \frac{H_z^{n+\frac{1}{2}}(i+\frac{1}{2}, j+\frac{1}{2}, k) - H_z^{n+\frac{1}{2}}(i+\frac{1}{2}, j-\frac{1}{2}, k)}{\Delta y} \\ & - \frac{H_y^{n+\frac{1}{2}}(i+\frac{1}{2}, j, k+\frac{1}{2}) - H_y^{n+\frac{1}{2}}(i+\frac{1}{2}, j, k-\frac{1}{2})}{\Delta z} \Bigg] \end{aligned}} \tag{11}$$

with

$$C_e = \frac{\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}, \qquad C_h = \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}.$$

Analogous formulas hold for E_y, E_z and for H_x, H_y, H_z using $\nabla \times \mathbf{E}$.

3.2 Boundary conditions: what we must enforce in PIC simulations

PIC simulations need boundaries for:

- **Open boundaries** (radiation into cladding/free space): PML is the default,
- **Metal boundaries** (PEC/finite conductivity), e.g., heaters, electrodes, package walls,
- **Symmetry boundaries** (mirror planes) to reduce simulation volume,
- **Periodic boundaries** (gratings, photonic crystals),
- **Ports/mode sources** (inject guided modes; measure S-parameters).

3.3 Perfect electric conductor (PEC) boundary (metal idealization)

For an ideal PEC surface:

$$\boxed{\text{Tangential } \mathbf{E} = 0 \quad \text{on the surface,} \qquad \text{Normal } \mathbf{B} = 0 \quad \text{on the surface.}} \tag{12}$$

Discrete enforcement in Yee FDTD: If a boundary coincides with a grid plane, set the tangential E components located on that boundary plane to zero each time step. Then H updates near the boundary naturally satisfy the curl relations with those E values. (For real metals at optical frequencies, one often uses dispersive material models instead of PEC.)

3.4 Perfect magnetic conductor (PMC) and symmetry boundaries

PMC is the dual boundary:

$$\text{Tangential } \mathbf{H} = 0, \qquad \text{Normal } \mathbf{D} = 0.$$

In practice, a symmetry plane can be represented by either PEC-like or PMC-like constraints depending on whether the field is odd or even across the plane. This allows halving or quartering the computational domain.

3.5 Absorbing boundaries and the need for PML

If you simply truncate the computational domain and impose (say) PEC or zero-field conditions, outgoing waves reflect back and contaminate results. Therefore we need an *absorbing boundary condition* (ABC) that mimics an infinite domain.

3.5.1 Berenger's PML idea

The **Perfectly Matched Layer (PML)** was introduced to absorb outgoing waves with (ideally) zero reflection at the interface with the interior domain. Berenger's original formulation used a **split-field** approach, splitting each field component into parts to create an anisotropic absorbing medium that is matched to the interior . Later formulations interpret PML through **complex coordinate stretching** (also described in PML literature and summarized in modern notes).

3.5.2 PML as an anisotropic lossy medium (conceptual derivation)

A simple lossy medium in frequency domain can be represented by a complex permittivity:

$$\tilde{\varepsilon}(\omega) = \varepsilon - \mathrm{i}\frac{\sigma}{\omega}, \qquad \tilde{\mu}(\omega) = \mu - \mathrm{i}\frac{\sigma_m}{\omega}, \tag{13}$$

which is explicitly written in standard PML lecture notes. However, a *uniform* lossy layer is not perfectly matched for oblique incidence in higher dimensions. PML constructs a specially designed anisotropic lossy region such that the tangential wave impedance is matched at the interface, eliminating reflections in the continuous limit.

3.5.3 Split-field PML (sketch with explicit split equation form)

To illustrate the split idea (2D example in many notes), a field component such as E_z is written as

$$E_z = E_{zx} + E_{zy}, \tag{14}$$

and separate conductivities σ_x, σ_y are applied to the split components, enabling matching conditions that cannot be achieved with a single isotropic conductivity. (Exact split equations depend on whether you are in TE_z or TM_z and on which components are present; the key is the direction-dependent loss profile.)

3.5.4 CPML remark (why it is popular)

Modern PMLs such as **convolutional PML (CPML)** avoid explicit field splitting and can be implemented with auxiliary differential equations / memory variables; many modern FDTD codes use CPML because it improves performance over a broad band and incidence angles.

3.6 What you should emphasize in class (practical PIC points)

- PML thickness and grading profile control reflections (discrete-world reflections are not exactly zero).
- Mesh resolution in high-index PIC cores is critical (numerical dispersion and staircase error).
- Metal boundaries: PEC is convenient for microwave-like problems; for optical metals one often needs dispersive Drude/Lorentz models (later lecture if needed).
- Boundary choice must be consistent with the physics: e.g., periodic for gratings, symmetry planes for modes, PML for radiation loss.

4 FDTD tutorial connection: Yee cell and the "FDTD 101" lecture series

A convenient companion to this lecture is the *FDTD 101* playlist that introduces the Yee grid, time stepping, and typical simulation workflows. Use it to visually connect:

- How the Yee cell stores field components,
- Why E and H are staggered in time (leapfrog),
- How PML surrounds the computational domain,
- How broadband spectra are computed from time-domain monitors.

5 Lumerical MODE: Finite-Difference Eigenmode (FDE) solver (math + connection)

5.1 Why eigenmodes matter in PIC

In PIC waveguides, the key objects are guided modes:

$$\mathbf{E}(x,y,z,t) = \mathrm{Re}\left\{\mathbf{E}(x,y)\,\mathrm{e}^{\mathrm{i}(\beta z-\omega t)}\right\}, \quad \mathbf{H}(x,y,z,t) = \mathrm{Re}\left\{\mathbf{H}(x,y)\,\mathrm{e}^{\mathrm{i}(\beta z-\omega t)}\right\}.$$

These are exactly the **monochromatic fields** (time harmonics) you derived previously, now specialized to waveguide invariance in z.

MODE's FDE solver computes:

- Mode profiles $\mathbf{E}(x,y), \mathbf{H}(x,y)$ on a cross-sectional mesh,

- Propagation constant β and effective index $n_{\text{eff}} = c\beta/\omega$,
- Loss (complex β) and dispersion via frequency sweeps.

The Ansys documentation explicitly states that MODE formulates Maxwell's equations into a **matrix eigenvalue problem** on a cross-sectional mesh and solves it using **sparse matrix techniques**.

5.2 Derive the eigenvalue problem from Maxwell (full-vector, waveguide invariance)

5.2.1 Start from frequency-domain Maxwell

Assume $e^{-i\omega t}$ time dependence (phasors). In source-free region:

$$\nabla \times \mathbf{E} = i\omega\mu\mathbf{H}, \tag{15}$$
$$\nabla \times \mathbf{H} = -i\omega\varepsilon\mathbf{E}. \tag{16}$$

Assume a waveguide invariant in z with fields

$$\mathbf{E}(x, y, z) = \mathbf{E}(x, y)\, e^{i\beta z}, \qquad \mathbf{H}(x, y, z) = \mathbf{H}(x, y)\, e^{i\beta z}. \tag{17}$$

Then $\partial/\partial z \mapsto i\beta$.

Define transverse gradient and curl operators:

$$\nabla_t = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y}, \qquad \mathbf{E} = \mathbf{E}_t + \hat{\mathbf{z}}E_z, \qquad \mathbf{H} = \mathbf{H}_t + \hat{\mathbf{z}}H_z.$$

5.2.2 Vector wave equation with β as eigenvalue

Take curl of (15):

$$\nabla \times \nabla \times \mathbf{E} = i\omega\mu(\nabla \times \mathbf{H}).$$

Insert (16):

$$\nabla \times \nabla \times \mathbf{E} = i\omega\mu(-i\omega\varepsilon\mathbf{E}) = \omega^2\mu\varepsilon\,\mathbf{E}.$$

Thus

$$\boxed{\nabla \times \nabla \times \mathbf{E} - \omega^2\mu\varepsilon\,\mathbf{E} = 0.} \tag{18}$$

Now insert the z-dependence (17). The operator ∇ includes $\partial_z = i\beta$. Therefore (18) becomes an equation on the **cross-section** (x, y) with parameter β. This can be rearranged into an eigenproblem of the form

$$\boxed{\mathbf{A}(\omega)\,\mathbf{e} = \beta^2\,\mathbf{B}(\omega)\,\mathbf{e},} \tag{19}$$

where $\mathbf{e}$ is a discretized representation of the transverse field unknowns on the mesh. (Depending on formulation, the eigenvalue can be β or β^2; Ansys notes that β^2 is the eigenvalue in their FDE formulation.)

Connection to your earlier Helmholtz results. In homogeneous media, (18) reduces to Helmholtz, and plane waves satisfy $k^2 = \omega^2\mu\varepsilon$. In a waveguide, the separation constant β plays the role of the longitudinal wavenumber, while transverse variation is set by $k_t^2 = k^2 - \beta^2$, matching the TE/TM slab derivations you already have.

5.3 Finite-difference discretization on the cross-section

5.3.1 Mesh and finite differences

On a rectangular mesh in (x, y):

$$x_i = x_0 + i\Delta x, \qquad y_j = y_0 + j\Delta y.$$

Centered finite differences:

$$\left.\frac{\partial f}{\partial x}\right|_{i,j} \approx \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x}, \qquad \left.\frac{\partial^2 f}{\partial x^2}\right|_{i,j} \approx \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{\Delta x^2},$$

and similarly for y. When applied to the coupled vector equations derived from (15)–(16) with $\partial_z = \mathrm{i}\beta$, each mesh point couples only to its nearest neighbors, producing a **sparse** matrix system.

5.3.2 Why the matrix is sparse (explicit neighbor coupling argument)

Each derivative operator uses only $f_{i\pm1,j}$ and $f_{i,j\pm1}$ (and possibly diagonals depending on the stencil). Therefore, each row of the discretized operator matrix has only $\mathcal{O}(1)$ nonzero entries, independent of the total number of mesh points. That is the structural reason sparse-matrix techniques are effective for FDE.

5.4 How MODE connects to this derivation (as stated by Ansys)

Ansys explicitly summarizes the MODE FDE workflow:

- Solve Maxwell's equations on a cross-sectional mesh with fields of the form $E(x, y)\mathrm{e}^{\mathrm{i}(-\omega t+\beta z)}$ and $H(x, y)\mathrm{e}^{\mathrm{i}(-\omega t+\beta z)}$,
- Formulate a matrix eigenvalue problem,
- Solve using sparse matrix techniques,
- Report $n_{\text{eff}} = c\beta/\omega$ and mode profiles,
- Use frequency sweeps for group delay/dispersion,
- Note that the eigenvalue is β^2 (can be negative/complex for evanescent or lossy modes) [?].

5.5 A minimal worked connection: slab TE/TM vs full-vector FDE

In the slab waveguide you derived scalar equations:

$$\frac{\mathrm{d}^2 E_y}{\mathrm{d}x^2} + \left(\omega^2\mu\varepsilon - \beta^2\right) E_y = 0 \quad \text{(TE)}, \qquad \frac{\mathrm{d}^2 H_y}{\mathrm{d}x^2} + \left(\omega^2\mu\varepsilon - \beta^2\right) H_y = 0 \quad \text{(TM)},$$

with boundary conditions involving $(1/\mu)\,\mathrm{d}E_y/\,\mathrm{d}x$ or $(1/\varepsilon)\,\mathrm{d}H_y/\,\mathrm{d}x$ continuity.

The MODE FDE solver generalizes this:

- from 1D (x only) to 2D (x, y),
- from scalar (single component) to **full-vector** (coupled components),
- from analytic boundary matching to numeric mesh-based boundary conditions.

Hence it can treat arbitrary PIC cross-sections (ridge/strip, slots, photonic crystal fibers, plasmonic guides), consistent with Ansys's statement that the finite-difference algorithm accommodates arbitrary waveguide structures and uses proprietary extensions (based on Zhu & Brown).

5.6 When to use FDTD vs MODE (conceptual summary)

- **MODE (FDE)**: best for *modal properties* of uniform waveguides (effective index, confinement, dispersion, bend modes).
- **FDTD**: best for *devices* that vary along propagation (gratings, couplers, MZIs with discontinuities, resonators, scattering, radiation).

The two are complementary: MODE gives accurate mode sources/ports for FDTD, and FDTD validates device performance.